Midterm

Exam Date and Time

Thursday, May 2, 1996. 2:20PM–4:50PM.

Note

This is a closed-book exam. However, you may consult the A4-sized sheet of notes that you prepared in advance. Each problem accounts for 10 points, unless otherwise marked.

Problems

1. Prove by induction that any tree can be colored with two colors such that each parent is in a different color from its children.

2. Construct a gray code of length $\lceil \log_2 18 \rceil$ $(= 5)$ for 18 objects. Show how the gray code is constructed from gray codes of smaller lengths.

3. Consider the following program segment in the celebrity algorithm.

```
i := 1;
j := 2;
next := 3;
while next <= n+1 do
   if Know[i,j] then i:= next
   else j := next;
   next := next + 1;
   end;
if i = n+1 then candidate := j
else candidate := i;
```

   (a) Find a loop invariant for the while loop that is sufficient to show that `candidate` will be the only possible candidate for the celebrity after the execution of the segment.
   (b) Prove that the loop invariant found above is indeed a loop invariant.

4. (a) What is the result of merging the following two skylines: (1,**8**,4,**11**,9,**0**,12,**6**,18,**15**,22) and (3,**7**,13,**4**,16,**10**,24). (5 points)
   (b) Give an algorithm for merging two skylines. (10 points)

5. In the towers of Hanoi puzzle, there are three pegs A, B, and C, with n (generalizing the original eight) disks of different sizes stacked in decreasing order on peg A. To objective is to transfer all the disks on peg A to peg B, moving one disk at a time (from one peg to one of the other two) and never having a larger disk stacked upon a smaller one.
   (a) Give an algorithm to solve the puzzle.
   (b) Compute the total number of moves in the algorithm.

6. (a) Apply the partition algorithm in quicksort to the following array (assuming that the first element is chosen as the pivot).

8	2	5	11	9	12	1	15	7	3	13	4	10	16	14	6

   Show the result after each exchange (swap) operation.

   (b) Apply the quicksort algorithm to the above array. Show the result after each partition operation.

7. (a) Rearrange the following array into a heap using the buttom-up approach.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	8	5	11	9	12	1	15	7	3	13	4	10	14	6

   Show the result after each element is added to the part of array that already satisfies the heap property. (5 points)

   (b) Give the buttom-up heap-building algorithm (in pseudo code). (10 points)

8. (a) Compute the *next* table (as in the KMP algorithm) for the string *ababcababdab*.
   (b) Modify the KMP algorithm to find the longest prefix of string B that matches a substring of A, assuming the *next* table for string B is given.

9. Given two strings *baa* and *bcba*, compute the minimal cost matrix $C[0..3, 0..4]$ for changing the first string character by character to the second one.