Midterm

Exam Date and Time

Tuesday, April 22, 1997. 2:20PM–5PM.

Note

This is a closed-book exam. Each problem accounts for 10 points, unless otherwise marked.

Problems

1. Prove by induction that the regions formed by a planar graph all of whose vertices have even degrees can be colored with two colors such that no two adjacent regions have the same color.

2. Prove by induction that a ring of even size can be colored with two colors and a ring of odd size with three colors such that no two adjacent nodes have the same color. (5 points)

3. Construct a gray code of length $\lceil \log_2 14 \rceil$ $(= 4)$ for 14 objects. Show how the gray code is constructed from gray codes of smaller lengths.

4. Let $b(n)$ denote the number of distinct binary trees with n nodes; for example, $b(1) = 1$, $b(2) = 2$, and $b(3) = 6$. We stipulate that $b(0) = 1$. Write a recurrence relation that defines $b(n)$, for $n \geq 0$. (5 points)

5. Show all intermediate and the final AVL trees formed by inserting the numbers from 9 down to 0.

6. For each of the following pairs of functions, say whether $f(n) = O(g(n))$ and/or $f(n) = \Omega(g(n))$. Justify your answers.

	$f(n)$	$g(n)$
(a)	$\frac{n^2}{\log n}$	$n(\log n)^2$
(b)	$n2^n$	3^n

7. (a) What is the result of merging the following two skylines: (1,**9**,3,**12**,9,**0**,12,**6**,18,**14**,22) and (3,**7**,13,**4**,16,**12**,21,**8**,25). (3 points)
(b) Give a detailed algorithm (in pseudo code) for merging two skylines. (7 points)

8. Apply the quicksort algorithm to the following array. Show the result after each partition operation.

8	1	5	11	16	12	2	15	7	3	13	4	10	9	14	6

9. Rearrange the following array into a heap using the buttom-up approach.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
8	2	5	11	9	12	3	10	7	1	13	4	15	14	6

Show the result after each element is added to the part of array that already satisfies the heap property.

10. Prove that the sum of the heights of all nodes in a complete binary tree with n nodes is at most $n-1$. (A complete binary tree with n nodes is one that can be compactly represented by an array A of size n, where the root is stored in $A[1]$ and the left and the right children of $A[i]$, $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$, are stored respectively in $A[2i]$ and $A[2i+1]$. Notice that, in Manber's book a complete binary tree is referred to as a balanced binary tree and a full binary tree as a complete binary tree. Manber's definitions seem to be less frequently used. Do not let the different names confuse you.)

11. Write a program (or modify the following code) to recover the solution to a knapsack problem using the *belong* flag. You should make your solution as efficient as possible.

Algorithm Knapsack (S, K);
begin
    $P[0,0].exist := true$;
    for $k := 1$ **to** K **do**
        $P[0,k].exist := false$;
    for $i := 1$ **to** n **do**
        for $k := 0$ **to** K **do**
            $P[i,k].exist := false$;
            if $P[i-1,k].exist$ **then**
                $P[i,k].exist := true$;
                $P[i,k].belong := false$
            else if $k - S[i] \geq 0$ **then**
                    if $P[i-1, k-S[i]].exist$ **then**
                        $P[i,k].exist := true$;
                        $P[i,k].belong := true$
end