Final

Exam Date and Time

Tuesday, June 10, 1997. 2:20PM–5PM.

Note

This is a closed-book exam. There are ten problems in total, each accounting for 10 points.

Problems

1. Apply the quicksort algorithm to the following array. Show the result after each partition operation; circle the element that was chosen as the pivot.

9	1	5	11	16	12	2	15	7	3	13	4	10	8	14	6

2. Rearrange the following array into a heap using the buttom-up approach.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
9	2	5	11	8	12	3	10	7	1	13	4	15	14	6

Show the result after each step that adds an element to the part of array that already satisfies the heap property; circle the elements that were exchanged in the step.

3. Explain why the time complexity of the KMP algorithm is $O(n)$, where n is the length of string A.

4. Given two strings *aabca* and *ccaba*, compute the minimal cost matrix $C[0..5, 0..5]$ for changing the first string character by character to the second one. Show the detail of your calculation for the entry $C[5, 5]$.

5. Prove that if the costs of all edges in a given connected graph are distinct, then the graph has an unique minimum-cost spanning tree.

6. Prove that Floyd's *All_Pairs_Shortest_Paths* algorithm works correctly for graphs with negative weights as long as there are no negative-weight cycles.

7. (a) Describe how the biconnected components of an undirected graph form a tree. (You should explain why the tree indeed does not contain any cycles.)
(b) Describe how the strongly connected components of a directed graph form a directed acyclic graph. (You should explain why the directed acyclic graph indeed does not contain any directed cycles.)

8. Let $G = (V, E)$ be a directed graph, and let T be a DFS tree of G. Prove that the intersection of the edges of T with the edges of any strongly connected component of G form a subtree of T.

9. The knapsack problem is as follows.

   Given a set X, where each element $x \in X$ has an associated size $s(x)$ and value $v(x)$, and two other numbers S and V, is there a subset $B \subseteq X$ whose total size is $\leq S$ and whose total value is $\geq V$?

   Prove that the knapsack problem is NP-complete.

10. The subgraph isomorphism problem is as follows.

    Given two graphs $G(V_1, E_1)$ and $H(V_2, E_2)$, does G contain a subgraph that is isomorphic to H? (Two graphs are isomorphic if there exists a one-one correspondence between the sets of vertices of the two graphs that preserve adjacency.)

    Prove that the subgraph isomorphism problem is NP-complete.

Appendix

- Floyd's algorithm for all-pairs shortest paths:

  Algorithm All_Pairs_Shortest_Paths (*weight*);
  begin
      for m := 1 to n **do**
          for x := 1 to n **do**
              for y := 1 to n **do**
                  if $weight[x, m] + weight[m, y] < weight[x, y]$ **then**
                      $weight[x, y] := weight[x, m] + weight[m, y]$
  end

- The partition problem: given a set X where each element $x \in X$ has an associated size $s(x)$, is it possible to partition the set into two subsets with exactly the same total size?
  The problem is NP-complete.

- The clique problem: given an undirected graph $G = (V, E)$ and an integer k, does G contain a clique of size $\geq k$?
  The problem is NP-complete.

- The Hamiltonian cycle problem: given a graph G, does G contain a Hamiltonian cycle? (A Hamiltonian cycle in a graph is a cycle that contains each vertex exactly once.)
  The problem is NP-complete.