Homework Assignment #7

Note

This assignment is due 2:10PM Tuesday, April 29, 2014. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points.

1. (6.16) Compute the *next* table as in the KMP algorithm for the string *aabaaabaab*. Show the details of your calculation.

2. (6.17) Given two strings *bacbb* and *baacab*, compute the minimal cost matrix $C[0..5, 0..6]$ for changing the first string character by character to the second one. Aside from giving the cost matrix, please show the details of how the entry $C[5, 6]$ is computed.

3. (6.39) Let A and B be two sets, both with n elements, such that A resides in computer P and B in computer Q. P and Q can communicate by sending messages, and they can perform any kind of local computation. Design an algorithm to find the nth smallest element (i.e., the median) of the union of A and B. You can assume, for simplicity, that all the elements are distinct. Your goal is to minimize the number of messages, where a message can contain one element or one integer. Please present your algorithm in an adequate pseudo code and make assumptions wherever necessary. What is the number of messages in the worst case?

4. (6.40) Design an algorithm that, given a set of integers $S = \{x_1, x_2, \ldots, x_n\}$, finds a nonempty subset $R \subseteq S$, such that

$$\sum_{x_i \in R} x_i \equiv 0 \pmod{n}.$$

   Before presenting your algorithm, please argue why such a nonempty subset must exist.

5. (6.62) You are asked to design a schedule for a round-robin tennis tournament. There are $n = 2^k$ players. Each player must play every other player, and each player must paly one match per round for $n - 1$ rounds. Denote the players by $P_1, P_2, \ldots, P_n$. Output the schedule for each player. (Hint: use divide and conquer in the following way. First, divide the players into two equal groups and let them play within the groups for the first $\frac{n}{2} - 1$ rounds. Then, design the games between the groups for the other $\frac{n}{2}$ rounds.)