Homework Assignment #3

Due Time/Date

2:20PM Tuesday, October 1, 2024. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please write or type your answers on A4 (or similar size) paper. Put your completed homework on the table in front of the instructor's desk before the class on the due date starts. For early or late submissions, please drop them in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (3.5) For each of the following pairs of functions, determine whether $f(n) = O(g(n))$ and/or $f(n) = \Omega(g(n))$. Justify your answers.

	$f(n)$	$g(n)$
(a)	$\frac{n}{\log n}$	$(\log n)^2$
(b)	$n^3 2^n$	3^n

2. Suppose f is a strictly increasing function that maps every positive integer to another positive integer, i.e., if $1 \le n_1 < n_2$, then $1 \le f(n_1) < f(n_2)$, and $f(n) = O(g(n))$ for some other function g. Is it true that $\log f(n) = O(\log g(n))$? Please justify your answer. How about $2^{f(n)} = O(2^{g(n)})$? Is it true?

3. Solve the following recurrence relation using *generating functions*. This is a very simple recurrence relation, but for the purpose of practicing you must use generating functions in your solution.

$$\begin{cases} T(1) = 1 \\ T(2) = 3 \\ T(n) = T(n-1) + 2\,T(n-2), \quad n \ge 3 \end{cases}$$

4. (3.26) Find the asymptotic behavior of the function $T(n)$ defined by the recurrence relation

$$\begin{cases} T(1) = 1 \\ T(n) = T(n/2) + \sqrt{n}, \quad n \ge 2. \end{cases}$$

You can consider only values of n that are powers of 2.

5. (3.30) Use Equation 1, shown below, to prove that $S(n) = \sum_{i=1}^{n} \lceil \log_2(n/i) \rceil$ satisfies $S(n) = O(n)$.

Bounding a summation by an integral
If $f(x)$ is a monotonically increasing continuous function, then

$$\sum_{i=1}^{n} f(i) \leq \int_{x=1}^{x=n+1} f(x) dx. \tag{1}$$