Midterm

(April 27, 2000)

Note

This is a closed-book exam. Each problem accounts for 10 points, unless otherwise marked.

Problems

1. (a) Consider a set $A = \{a, b, c, d, e\}$ and a relation $R = \{(a, b), (a, c), (d, e)\}$ on A. Find the smallest equivalence relation on A that contains R.

   (b) Suppose that R_1 and R_2 are equivalence relations on a set A. Is $R_1 \cup R_2$ an equivalence relation on A? Justify your answer.

2. (a) Draw the state diagram of an NFA, with as few states as possible, that recognizes $\{w \in \{0,1\}^* \mid w \text{ ends with 01 or 10}\}$. (5 points)

   (b) Convert the NFA in (a) into an equivalent DFA (using the procedure discussed in class); do not attempt to optimize the number of states. (10 points)

3. (a) Draw the state diagram of a DFA that recognizes $A = \{w \in \{0,1\}^* \mid w \text{ doesn't contain the substring } 011\}$.

   (b) Translate the DFA in (a) to an equivalent context-free grammar (using the procedure discussed in class).

4. Translate the DFA in Problem 3 to an equivalent regular expression (using the procedure discussed in class). Please show the intermediate automata.

5. Give a context-free grammar for generating $\{w\#x \mid w^{\mathcal{R}} \text{ is a substring of } x \text{ for } w, x \in \{0,1\}^*\}$. Please explain the intuition behind the grammar.

6. Convert the following CFG into an equivalent CFG in Chomsky normal form (using the procedure discussed in class).

$$S \rightarrow (S) \mid SS \mid \varepsilon$$

Note: There are various definitions of Chomsky normal form. Below is the definition that we used in class.

A context-free grammar is in **Chomsky normal form** if every rule is of the form

$$\begin{array}{rcl} A & \to & BC \quad \text{or} \\ A & \to & a \end{array}$$

where a is any terminal and A, B, and C are any variables—except that B and C may not be the start variable. In addition,

$$S \to \varepsilon$$

is permitted if S is the start variable.

7. Draw the state diagram of a pushdown automaton that recognizes the set of strings over $\{a, b\}$ with twice as many a's as b's. Please explain the intuition behind the automaton.

8. (a) Prove that the class of context-free languages is closed under the regular operations: *union*, *concatenation*, and *star*. (Hint: A language is context-free if it is generated by some context-free grammar.) (10 points)

   (b) Prove that the class of context-free languages is not closed under either *intersection* or *complement*. (Hint: Find two languages that are context-free, but their intersection is not.) (5 points)

9. Prove, using the pumping lemma, that $\{a^p \mid p \text{ is a prime number}\}$ is not context-free.