Final

(June 20, 2002)

Note

This is a closed-book exam. Each problem accounts for 10 points, unless otherwise marked.

Problems

1. Draw the state diagram of a pushdown automaton that recognizes the language $\{w\#x \mid w^{\mathcal{R}} \text{ is a substring of } x \text{ for } w, x \in \{a,b\}^*\}$. Explain the intuition behind the automaton by showing how it accepts the input $ab\#abab$. (15 points)

2. Give the implementation-level description of a (single-tape deterministic) Turing machine that decides the language in Problem 1.

3. Briefly explain why a pushdown automaton with three stacks are not more powerful (recognizing a larger class of languages) than one with two stacks. (5 points)

4. Prove that a language is decidable if and only if some enumerator enumerates the language in lexicographic order.

5. Prove that, for any countable set A, there cannot exist a (one-to-one) correspondence between A and 2^A (the power set of A).

6. Let $SUBSET_{\mathrm{DFA}} = \{\langle D_1, D_2\rangle \mid D_1 \text{ and } D_2 \text{ are DFAs and } L(D_1) \subseteq L(D_2)\}$. Show that $SUBSET_{\mathrm{DFA}}$ is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works.

7. Prove that EQ_{CFG} is not Turing-recognizable.

8. Show that if A is Turing-recognizable and $A \leq_m \overline{A}$, then A is decidable.

9. In the proof of the Cook-Levin theorem, which states that SAT is NP-complete, we used 2×3 windows of cells to formulate the constraint that the configuration of each row (except the first one) in the $n^k \times n^k$ tableau follows legally from the configuration of the preceding row. Why couldn't we use two entire rows of cells directly?

10. Let $TRIPLE\_SAT = \{\langle\phi\rangle \mid \phi$ has at least three satisfying assignments$\}$. Prove that $TRIPLE\_SAT$ is NP-complete.

Appendix

- $A_{\mathrm{TM}} = \{\langle M, w\rangle \mid M$ is a TM and M accepts $w\}$. A_{TM} is undecidable.
- $EQ_{\mathrm{CFG}} = \{\langle G, H\rangle \mid G$ and H are CFGs and $L(G) = L(H)\}$. EQ_{CFG} is undecidable.
- A language is decidable if and only if it is both Turing-recognizable and co-Turing-recognizable.
- $A \leq_m B$ is equivalent to $\overline{A} \leq_m \overline{B}$.
- $SAT = \{\langle\phi\rangle \mid \phi$ is a satisfiable Boolean formula$\}$. SAT is NP-complete.