Midterm

Note

This is a closed-book exam. Each problem accounts for 10 points, unless otherwise marked.

Problems

1. (a) Give a binary relation on $A = \{a, b, c, d\}$ that is symmetric and transitive but not reflexive.

   (b) Let $R = \{(a, c), (b, c), (d, e)\}$ be a binary relation on $A = \{a, b, c, d, e\}$. Find the smallest equivalence relation on A that contains R.

2. (a) Draw the state diagram of an NFA, with as few states as possible, that recognizes $\{w \in \{0, 1\}^* \mid w \text{ ends with 00, 01, or 11}\}$. The fewer states your NFA has, the more points you will be credited for this problem. (5 points)

   (b) Convert the NFA in (a) systematically into an equivalent DFA (using the procedure discussed in class); do not attempt to optimize the number of states. (10 points)

3. (a) Draw the state diagram of a DFA, with as few states as possible, that recognizes $\{w \in \{0, 1\}^* \mid w \text{ doesn't contain 000 or 010 as a substring}\}$. The fewer states your DFA has, the more points you will be credited for this problem. (10 points)

   (b) Translate the DFA in (a) systematically to an equivalent context-free grammar (using the procedure discussed in class). (5 points)

4. Write a regular expression for the language in Problem 3.

5. Convert the following CFG into an equivalent CFG in Chomsky normal form (using the procedure discussed in class: add a new start symbol, remove illegal ε rules, remove unit rules, ...).

$$\begin{array}{rcl} A & \rightarrow & BAB \mid B \mid 11 \\ B & \rightarrow & 00 \mid \varepsilon \end{array}$$

6. Draw the state diagram of a pushdown automaton (PDA) that recognizes the following language: $\{w\#x \mid w^{\mathcal{R}} \text{ is a substring of } x \text{ for } w, x \in \{0,1\}^*\}$. Please explain the intuition behind the PDA.

7. Prove, using the pumping lemma, that $\{a^p \mid p \text{ is a prime number}\}$ is not context-free.

8. Prove that the language over $\{a, b, c\}$ with equal numbers of a's, b's, and c's is not context-free.

9. Find a regular language A, a non-regular but context-free language B, and a non-context-free language C over $\{a, b\}$ such that $C \subseteq B \subseteq A$.

Appendix

- Properties of a binary relation R:
  - R is *reflexive* if for every x, xRx.
  - R is *symmetric* if for every x and y, xRy if and only if yRx.
  - R is *transitive* if for every x, y, and z, xRy and yRz implies xRz.

- A context-free grammar is in **Chomsky normal form** if every rule is of the form

$$\begin{array}{lll} A & \rightarrow & BC \quad \text{or} \\ A & \rightarrow & a \end{array}$$

  where a is any terminal and A, B, and C are any variables—except that B and C may not be the start variable. In addition,

$$S \rightarrow \varepsilon$$

  is permitted if S is the start variable.

- If A is a regular language, then there is a number p (the pumping length) such that, if s is any string in A and $|s| \geq p$, then s may be divided into three pieces, $s = xyz$, satisfying the conditions: (1) for each $i \geq 0$, $xy^iz \in A$, (2) $|y| > 0$, and (3) $|xy| \leq p$.

- If A is a context-free language, then there is a number p such that, if s is a string in A and $|s| \geq p$, then s may be divided into five pieces, $s = uvxyz$, satisfying the conditions: (1) for each $i \geq 0$, $uv^ixy^iz \in A$, (2) $|vy| > 0$, and (3) $|vxy| \leq p$.

- The language $\{a^nb^nc^n \mid n \geq 0\}$ is not context-free.

- The intersection of a context-free language and a regular language is context-free.