Final

Note

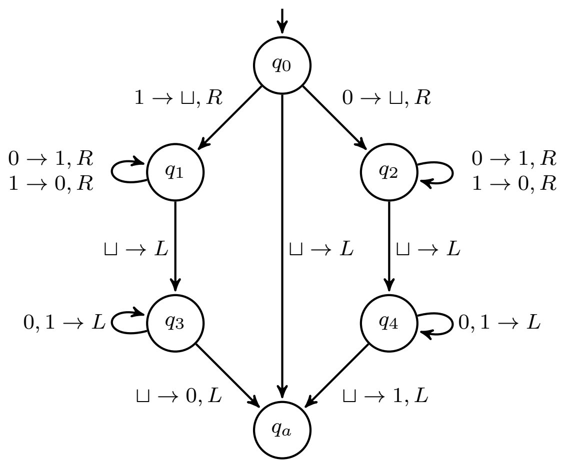
This is an open-book exam. You may consult any book, paper, note, or on-line resource, but discussions with others (in person or via a network) are strictly forbidden.

How to Submit Your Answers

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b067050xx-final". Upload the PDF file by the due time to the NTU COOL course site for Theory of Computing 2021.

Problems

1. Below is a formal description of a Turing machine that computes a function from $\{0,1\}^*$ to $\{0,1\}^*$. Explain in words what exactly the machine computes.



111000111001101
1111000111001101

2. Give a formal description of a (single-tape deterministic) Turing machine that computes a function from $\{0,1\}^*$ to $\{0,1\}^*$. It appends a 0 at the front of the input if the input starts with a 0 and it appends a 1 otherwise. It produces a 0 for the empty string. The machine should stop with the tape head pointing at the left most cell of the tape.

3. Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.

4. A *useless state* in a pushdown automaton is a state that is never entered on any input. Show the decidability of the problem of determining whether a given pushdown automaton has a useless state.

A' \union B' = \Sigma*

5. Let A and B be two disjoint languages. Say that language C *separates* A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language. $M_{A'}$ $M_{B'}$

6. Discuss the applicability of Rice's Theorem for each of the following problems (languages). Please give the reasons why or why not.

   (a) $\{\langle M \rangle \mid M \text{ is a TM and } 101 \notin L(M)\}$.

   (b) $COUNTABLE_{\mathrm{TM}} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is countable}\}$.

A' <= A_

7. Show that if A is Turing-recognizable and $A \leq_m \overline{A}$, then A is decidable.

8. In the proof of the Cook-Levin theorem, which states that SAT is NP-complete, we used 2×3 windows of cells to formulate the constraint that the configuration of each row (except the first one) in the $n^k \times n^k$ tableau legally follows the configuration of the preceding row. Suppose the Turing machine being reduced is that in Problem 1.

   Which of the following 2×3 windows of cells are illegal? Why?

q_0	1	0
$\sqcup$	q_2	0

1	q_2	0
1	1	q_2

0	1	0
1	0	1

$\sqcup$	1	0
$\sqcup$	1	0

1	1	0
1	1	q_3

q_a	0	1
q_a	0	1

9. In the proof that the *3SAT* problem is polynomially reducible to the *CLIQUE* problem, we convert an arbitrary Boolean expression in 3CNF (input of the *3SAT* problem) to a graph and an integer (input of the *CLIQUE* problem).

   (a) Please illustrate the conversion by drawing the graph and giving the integer that will be obtained from the following boolean expression:

$$(\overline{x} + y + z) \cdot (\overline{w} + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{w} + x + \overline{y}).$$

   (b) The original Boolean expression is satisfiable. As a demonstration of why the reduction is correct, please use the obtained result to argue that it is indeed the case.

x \in 3SAT iff f(x) \in CLIQUE

10. In the proof that the *SAT* problem is polynomially reducible to the *3SAT* problem, we convert an arbitrary Boolean expression in CNF (input of the *SAT* problem) to another in 3CNF (input of the *3SAT* problem).

   (a) Please illustrate the conversion by giving the Boolean expression in 3CNF that will be obtainen from the following Boolean expression:

$$(w + \overline{x} + y + z) \cdot (v + \overline{w} + x + \overline{y} + \overline{z}) \cdot (\overline{v} + y).$$

   (b) The original Boolean expression is satisfiable. As a demonstration of why the reduction is correct, please use the resulting expression to argue that it is indeed the case.

(w + x' + a) . (a' + y + z) . ….

(v' + y) = (v' + y + b) (v' + y + b') = (v' + y + y)

(x) = (x + x + x)