Final

Note

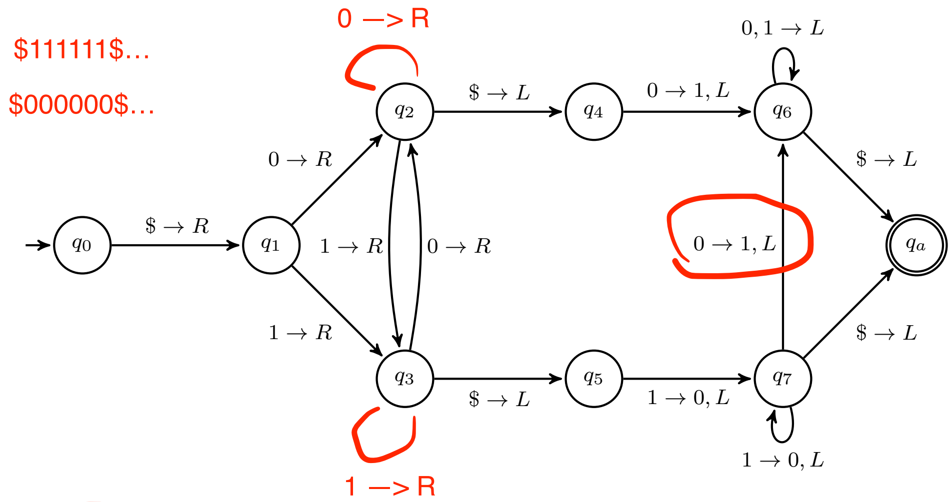
This is an open-book exam. You may consult any book, paper, note, or on-line resource, but discussions with others (in person or via a network) are strictly forbidden.

How to Submit Your Answers

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b077050xx-final". Upload the PDF file by the due time to the NTU COOL course site for Theory of Computing 2022.

Problems

1. Below is a formal description of a Turing machine that computes a function from $\{0, 1, \$\}^*$ to $\{0, 1, \$\}^*$. The machine is meant to be deterministic; however, for brevity we have omitted transitions that go to a rejecting and terminating state. Explain in words what exactly the machine computes. You only need to consider the cases where the computation successfully terminates at state q_a.



2. Give a formal description of a (single-tape deterministic) Turing machine that decides the language $\{1^k \# 1^{k^2} \mid k \geq 1\}$.

3. Let $C = \{\langle G, x\rangle \mid G$ is a CFG and x is a substring of some $y \in L(G)\}$. Show that C is decidable. (Hint: an elegant solution to this problem uses the decider for E_{CFG}.)

L(G) \intersect \Sigma* x \Sigma*

4. Let A and B be two disjoint languages. Say that language C *separates* A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

\bar{A} \union \bar{B} = \Sigma*

L(M) separates A and B

L(M) separates B and A

5. Let $AMBIG_{\text{CFG}} = \{\langle G \rangle \mid G \text{ is an ambiguous CFG}\}$. Show that $AMBIG_{\text{CFG}}$ is undecidable. (Hint: use a reduction from PCP. Given an instance

$$P = \left\{ \left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots, \left[\frac{t_k}{b_k}\right] \right\}$$

of PCP, construct a CFG G with the rules:

1 2 3 . . n   1 2 3 . . n

$$\begin{aligned} S &\to T \mid B \\ T &\to t_1 T a_1 \mid \cdots \mid t_k T a_k \mid t_1 a_1 \mid \cdots \mid t_k a_k \\ B &\to b_1 B a_1 \mid \cdots \mid b_k B a_k \mid b_1 a_1 \mid \cdots \mid b_k a_k, \end{aligned}$$

where $a_1, \ldots, a_k$ are new terminal symbols. Prove that this reduction works.)

6. Two graphs G and H are said to be *isomorphic* if the nodes of G may be renamed so that it becomes identical to H. Let $ISO = \{\langle G, H \rangle \mid G \text{ and } H \text{ are isomorphic}\}$. Prove that $ISO \in \text{NP}$, using the definition $\text{NP} = \bigcup_k \text{NTIME}(n^k)$. Please be precise about the steps in a nondeterministic selection and make sure that it can be done in polynomial time.

7. In the proof of the Cook-Levin theorem, which states that SAT is NP-complete, we used 2×3 windows of cells to formulate the constraint that the configuration of each row (except the first one) in the $n^k \times n^k$ tableau legally follows the configuration of the preceding row. Suppose the Turing machine being reduced is that in Problem 1 (though it is a function computer rather than a language decider).

Which of the following 2×3 windows of cells are illegal? Why?

C1 yields C2

a q_2 $ —> q_4 a $

q_0	\$	0
\$	0	q_2

1	q_2	\$
q_4	1	\$

q_3	\$	0
1	\$	0

\$	1	0
\$	1	q_6

0	1	0
1	1	0

q_7	\$	1
q_a	\$	1

|C|^6

8. Following the previous problem, why couldn't we simply use two entire rows of cells to formulate the constraint?

|C|^{2n^k}

9. In the proof that the $3SAT$ problem is polynomially reducible to the $CLIQUE$ problem, we convert an arbitrary Boolean expression in 3CNF (input of the $3SAT$ problem) to a graph and an integer (input of the $CLIQUE$ problem).

   (a) Please illustrate the conversion by drawing the graph and giving the integer that will be obtained from the following boolean expression:

$$(\overline{w} + y + z) \cdot (\overline{x} + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (w + \overline{x} + \overline{y}).$$

   (b) The original Boolean expression is satisfiable. As a demonstration of why the reduction is correct, please use the obtained result to argue that it is indeed the case.

w \in A iff f(w) \in B

10. In the proof that the *SAT* problem is polynomially reducible to the *3SAT* problem, we convert an arbitrary Boolean expression in CNF (input of the *SAT* problem) to another in 3CNF (input of the *3SAT* problem).

   (a) Please illustrate the conversion by giving the Boolean expression in 3CNF that will be obtainen from the following Boolean expression:

   $$(v + \overline{y}) \cdot (\overline{v} + \overline{w} + x + y + \overline{z}) \cdot (w + \overline{x} + \overline{y} + z).$$

   (b) The original Boolean expression is satisfiable. As a demonstration of why the reduction is correct, please use the resulting expression to argue that it is indeed the case.

A <= B

A <=_m B   w \in A iff f(w) \in B

A <=_p B   w \in A iff f(w) \in B