Homework Assignment #1

Note

This assignment is due 2:10PM Tuesday, March 17, 2020. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 0.7; 30 points) For each part, give a binary relation that satisfies the condition. *Please illustrate the relation using a directed graph.*
   (a) Reflexive and symmetric but not transitive
   (b) Reflexive and transitive but not symmetric
   (c) Symmetric and transitive but not reflexive
2. (20 points) For each part, determine whether the binary relation on the set of integers is an equivalence relation. If it is, please provide a proof; otherwise, please give a counterexample.
   (a) The two numbers have a common divisor.
   (b) For a fixed non-zero divisor, the two numbers have the same remainder. (Note: for instance, suppose 2 is the divisor. Numbers 4 and 6 have the same remainder, while 4 and 5 do not.)
3. (20 points) In class, following Sipser's book, we first studied the formal definition of a function and then treated relations as special cases of functions. Please give instead a direct definition of relations and then define functions as special cases of relations. Your definitions should cover the arity of a relation or function and also the meaning of the notation $f(a) = b$.
4. (Problem 0.10; 20 points) Show that every graph having two or more nodes contains two nodes with the same degree.
5. (Problem 0.11; 10 points) Find the error in the following proof that all horses are the same color.

   CLAIM: In any set of h horses, all horses are the same color.

   PROOF: By induction on h.

   Basis ($h = 1$): In any set containing just one horse, all horses clearly are the same color.

Induction step ($h > 1$): We assume that the claim is true for $h = k$ ($k \geq 1$) and prove that it is true for $h = k + 1$. Take any set H of $k + 1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2. By the same argument, all the horses in H_2 are the same color. Therefore all the horses in H must be the same color, and the proof is complete.