Homework Assignment #10

Due Time/Date

This assignment is due 2:10PM Tuesday, June 9, 2020. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b057050xx-hw10". Upload the PDF file to the Ceiba course site for Theory of Computing 2020: https://ceiba.ntu.edu.tw/1082theory2020. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 5.9; 10 points) Let $AMBIG_{\mathrm{CFG}} = \{\langle G \rangle \mid G$ is an ambiguous CFG$\}$. Show that $AMBIG_{\mathrm{CFG}}$ is undecidable. (Hint: use a reduction from PCP. Given an instance

$$P = \left\{ \left[\frac{t_1}{b_1} \right], \left[\frac{t_2}{b_2} \right], \cdots, \left[\frac{t_k}{b_k} \right] \right\}$$

   of PCP, construct a CFG G with the rules:

$$
\begin{aligned}
S &\rightarrow T \mid B \\
T &\rightarrow t_1 T a_1 \mid \cdots \mid t_k T a_k \mid t_1 a_1 \mid \cdots \mid t_k a_k \\
B &\rightarrow t_1 B a_1 \mid \cdots \mid t_k B a_k \mid t_1 a_1 \mid \cdots \mid t_k a_k,
\end{aligned}
$$

   where $a_1, \ldots, a_k$ are new terminal symbols. Prove that this reduction works.)

2. (Problem 5.14(b); 20 points) Define a *two-headed finite automaton* (2DFA) to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-end and one on the right-hand end, that serve as delimiters. A 2DFA accepts its input by entering a special accept state. For example, a 2DFA can recognize the language $\{a^n b^n c^n \mid n \geq 0\}$.

   Let $E_{2\mathrm{DFA}} = \{\langle M \rangle \mid M$ is a 2DFA and $L(M) = \emptyset\}$. Show that $E_{2\mathrm{DFA}}$ is undecidable.

3. (Problem 5.18(b); 10 points) Use Rice's theorem to prove the undecidability of the language $\{\langle M \rangle \mid M$ is a TM and $101 \in L(M)\}$. (Note: you should show that Rice's theorem is applicable for the problem/language.)

4. (Problem 5.22; 20 points) Let $X = \{\langle M, w \rangle \mid M$ is a single-tape TM that never modifies the portion of the tape that contains the input $w\}$. Is X decidable? Prove your answer.

5. (Problem 5.29; 20 points) A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

6. (Problem 5.36; 20 points) Prove that there exists an undecidable subset of $\{1\}^*$.