Homework Assignment #1

Due Time/Date

This assignment is due 2:20PM Tuesday, March 1, 2022. Late submission will be penalized by 20% for each working day overdue.

Note

Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 0.7; 30 points) For each part, give a binary relation that satisfies the condition. *Please illustrate the relation using a directed graph.*

   (a) Reflexive and symmetric but not transitive

   (b) Reflexive and transitive but not symmetric

   (c) Symmetric and transitive but not reflexive

2. (20 points) For each part, determine whether the binary relation on the set of reals or integers is an equivalence relation. If it is, please provide a proof; otherwise, please give a counterexample.

   (a) The two numbers have a common divisor.

   (b) For a fixed non-zero divisor, the two numbers have the same remainder. (Note: for instance, suppose 2 is the divisor. Numbers 4 and 6 have the same remainder, while 4 and 5 do not.)

3. (20 points) In class, following Sipser's book, we first studied the formal definition of a function and then treated relations as special cases of functions. Please give instead a direct definition of relations and then define functions as special cases of relations. Your definitions should cover the arity of a relation or function and also the meaning of the notation $f(a) = b$.

4. (Problem 0.10; 20 points) Show that every graph having two or more nodes contains two nodes with the same degree. (Note: we assume that every graph is simple and finite, unless explicitly stated otherwise.)

5. Consider a round-robin tournament among n players. In the tournament, each player plays once against all other $n-1$ players. There are no draws, i.e., for a match between p and p', the result is either p beat p' or p' beat p. Prove *by induction* that, after a round-robin tournament, it is always possible to arrange the n players in an order $p_1, p_2, \ldots, p_n$ such that p_1 beat p_2, p_2 beat p_3, $\cdots$, and p_{n-1} beat p_n. (Note: the "beat" relation, unlike "$\geq$", is not transitive.)