Homework Assignment #2

Due Time/Date

This assignment is due 2:20PM Tuesday, March 12, 2024. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file and name the file according to this pattern: "b107050xx-hw2". Upload the PDF file to the NTU COOL site for this course. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 1.3 adapted; 10 points) The formal definition of a DFA M is $(\{q_1, q_2, q_3, q_4, q_5\}, \{\texttt{a}, \texttt{b}\}, \delta, q_3, \{q_3\})$ where δ is given by the following table. Draw the state diagram of M and give an intuitive characterization of the strings that M accepts.

	a	b
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

2. (Exercise 1.4; 20 points) Each of the following languages is the intersection of two simpler regular languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in class (see also Footnote 3 in Page 46 of [Sipser 2006, 2013]) to give the state diagram of a DFA for the language given. In all parts, the alphabet is $\{\texttt{a}, \texttt{b}\}$.
   (a) $\{w \mid w$ starts with an $\texttt{a}$ and has at most one $\texttt{b}\}$.
   (b) $\{w \mid w$ has an odd number of $\texttt{a}$'s and ends with a $\texttt{b}\}$.

3. (Exercise 1.6; 20 points) Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{\texttt{0}, \texttt{1}\}$.
   (a) $\{w \mid w$ doesn't contain the substring $\texttt{110}\}$.
   (b) $\{w \mid$ every odd position of w is a $\texttt{1}\}$ (Note: see w as $w_1 w_2 \cdots w_n$, where $w_i \in \{\texttt{0}, \texttt{1}\}$).

4. (Problem 1.36; 10 points) For any string $w = w_1 w_2 \cdots w_n$, the *reverse* of w, written w^R, is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that, if A is regular, so is A^R.

5. (Problem 1.37; 20 points) Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Σ_3 contains all size 3 columns of `0`s and `1`s. A string of symbols in Σ_3 gives three rows of `0`s and `1`s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \text{but} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that B is regular. (Hint: working with B^R is easier. You may assume the result claimed in the previous problem (Problem 1.36).)

6. (20 points) Generalize the proof of Theorem 1.25 of [Sipser 2006, 2013] (Pages 45–47) to handle A_1 and A_2 with different alphabets.